The Prisoner’s Dilemma and Repeated Games

Games Of Strategy
Chapter 10
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Terms to Know

- Compound Interest
- Contingent Strategy
- Discount Factor
- Effective rate of Return
- Grim Strategy
- Infinite Horizon
- Leadership

- Penalty
- Present Value
- Punishment
- Repeated Play
- Tit-For-Tat
- Trigger Strategy
Introductory Game 1

- Line up based on the fourth to the last letter of your first name; ties are decided by a game of rock-paper-scissors where the winner is on the right of the loser
- Count off from 1 to the number of individuals in the class (N)
- Person 1 will pair up with person N, person 2 will pair up with person N-1, etc.
Introductory Game 1 Cont.

- Each person will go to opposite sides of the room.
- The professor will hand you a discount factor that ranges from 1 to 20 where the number represents your discount rate for each stage of the game.
- The professor will also give you two pieces of paper where you will write cooperate on one side and not cooperate on the other side on one piece and you will use the second piece to keep track of your decisions, your opponents decisions, and your scores in each round.
Introductory Game 1 Cont.

- When the professor says show your decision, you must hold up your decision immediately
- Write down the round and your discounted score
- Your payoff matrix for the first round which is not discounted is on the next slide
Introductory Game 1 Cont.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Cooperate</th>
<th>Don't Cooperate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>10, 10</td>
<td>2, 18</td>
</tr>
<tr>
<td>Don’t Cooperate</td>
<td>18, 2</td>
<td>9, 9</td>
</tr>
</tbody>
</table>
Introductory Game 1 Cont.

- The game will continue until the professor chooses to stop it (note it may go on for awhile)
- Once the professor says stop, tally up your score and hand it in
Discussion

- How many individuals were able to cooperate through the whole game? Why?
- How many individuals were unable to cooperate through the whole game? Why?
- What strategy did you employ when playing the game?
- Did your discount factor have an effect on your decision?
Discussion Cont.

- Did the end of the game have an effect on the strategy you chose?
- How can this be related to an agribusiness scenario?
- Other thoughts?
Basic Review of Infinite Sums

- Define $\delta = 1/(1+r) < 1$, then we know that:
  - $\sum_{n=1}^{\infty} \delta^n = \frac{\delta}{1-\delta} = \frac{1}{r}$ and
  - $\sum_{n=1}^{m} \delta^n = \frac{\delta - \delta^{m+1}}{1-\delta}$

- E.g., $\frac{1}{2} + 1/4 + 1/8 + \ldots = 1$
- E.g., $\frac{1}{2} + 1/4 + 1/8 + 1/16 = 15/16$
Discounting a Lump Sum

- **Problem:** Suppose you wanted to know what is the present value of a sum to be received in the future?

- **Formula:** $PV = \frac{FV}{(1 + i)^n}$
  - Where $PV$ = the current value of a future amount at the end of $n$ periods
  - $FV$ = future amount
  - $i$ = compound interest rate and
  - $n$ = number of periods.
Sustaining Cooperation in the Prisoner’s Dilemma

- Repeated play of the prisoner’s dilemma game can lead to cooperation
- If the future rewards of long-term cooperation can outweigh the short-term value of defection, cooperation can be upheld in a prisoner’s dilemma game
- This idea brings in the need for discounting future value to bring them to present value
Finitely Repeated Games with a Known Ending Time

- When finitely repeated prisoner’s dilemma games are played and the ending date is known, rollback analysis would dictate that the players should not cooperate

- Why?
Infinite Repetition of Prisoner’s Dilemma

- If games will go on for an infinite period, then it is possible that cooperation will hold up in repeated prisoner’s dilemma’s.
- Under this scenario, it is incumbent on the player to analyze the value of defection (do not cooperate) along with the value of the infinite stream from staying with cooperation.
Contingent Strategies

- **Trigger Strategies**
  - With these strategies, once a player defects, the strategy calls for punishing the defector through non-cooperation for a specified time period.
  - This strategy allows for forgiveness.

- **Grim Strategy**
  - This strategy dictates that once a competitor doesn’t cooperate, then you should not cooperate ever again.
Contingent Strategies Cont.

- Tit-For-Tat Strategy
  - This strategy calls for you to follow what the other player does; if the individuals cooperate then you cooperate, otherwise if they do not cooperate, then do not cooperate.
  - This strategy is a powerful strategy because it also allows for forgiveness.
Calculating the Value of Defection from the Introductory Game at $r = 10\%$

- Assuming a grim strategy, defection gains the individual 8 but he loses out on the value of cooperation which is 1 for every period thereafter
  - Is it worth it?
- Need to calculate the PV of an infinite sum of 1’s with a discount rate of 10\%
Calculating the Value of Defection Cont.

- This value would be $1/r = 1/.1 = 10$
- Since the present value of the infinite sum is greater, then cooperation should continue.
- What discount rate would make you indifferent between defection and cooperation?
- $8 = 1/r$ or $r = 1/8 = 12.5\%$
Calculating the Value of Defection Cont.

- Assuming a tit-for-tat strategy of defection and then cooperation, defection gains the individual 8 but he loses out on 8 for the next period
  - Is it worth it?
- Need to calculate the PV of 8 with a discount rate of 10%
Calculating the Value of Defection Cont.

- This value would be $8/(1+r) = 8/1.1 = 7.27$
- Since the present value of the discounted value is less than the gain from not cooperating, the individual should defect then cooperate.
- What discount rate would make you indifferent between defection and cooperation?
Calculating the Value of Defection Cont.

- $8 = \frac{8}{1+r}$ or $r = 0$, i.e., there should be no discounting
- How would this change with a trigger strategy of a 2 period punishment?
Finite Games with A Probabilistic Ending Time

- When you only know the probability that the game will end you need to calculate the effective rate of return \( R \)
  \[
  R = \frac{(1-p\delta)}{p\delta}
  \]
  where \( p \) is the probability the game will continue and \( \delta = \frac{1}{1+r} \)
- As \( p \) decreases \( R \) will increase making defection more likely
Other Solutions to the Prisoner’s Dilemma

- Penalties
  - An individual can change the payoff structure by imposing a large penalty, e.g., matching the best offer plus 10%.

- Leadership
  - This can occur when the payoffs of the players are vastly different and one player finds it worthwhile to cooperate even when the other chooses not to.
Final Discussion, Questions, and Thoughts