Combining Sequential and Simultaneous Moves

Games Of Strategy
Chapter 6
Dixit, Skeath, and Reiley
Terms to Know

- Continuation
- Credibility
- Information Set
- Off-Equilibrium Path
- Off-Equilibrium Subgame
- Subgame
- Subgame-Perfect Equilibrium
Introductory Game

- Line up in alphabetical order based on your first name from A to Z
- Count off from 1 to the number of individuals in the class which we will call N
- If you are 1 through N/2, your partner will be the individual who is N/2 more than your number, e.g., suppose there are 20 people in the class, the person who is 1 will be paired up with the person who is 11
Introductory Game Cont.

- Select an individual who will be going first by using rock-paper-scissors (best out of three), the person who wins gets to decide whether he/she wants to go first or second.

- Suppose player 1 is an early adopter farmer when technology is concerned, while player 2 is a second farmer who examines what farmer 1 does and then makes his decision on what technology to adopt.
Introductory Game Cont.

- Each farmer has the choice of choosing one of two different technologies—Drones or Precision Tractors.
- The first farmer will write down on a piece of paper his choice of technology and show it proudly to the other farmer.
The second farmer will choose one of the two technologies and write it down on a piece of paper but will not show it to the other person.

Each farmer will take a second piece of paper and write down whether the individual wants to produce tomatoes or potatoes and not show it to the other until the professor says so.
Introductory Game Cont.

- If **Farmer 1** chooses to use a **Drone** to produce **Tomatoes** and **Farmer 2** uses a **Drone** to produce **Tomatoes**, **Farmer 1** will receive 45 and **Farmer 2** will receive 45.

- If **Farmer 1** chooses to use a **Drone** to produce **Potatoes** and **Farmer 2** uses a **Drone** to produce **Tomatoes**, **Farmer 1** will receive 40 and **Farmer 2** will receive 60.
Introductory Game Cont.

- If Farmer 1 chooses to use a Drone to produce Tomatoes and Farmer 2 uses a Drone to produce Potatoes, Farmer 1 will receive 60 and Farmer 2 will receive 40.

- If Farmer 1 chooses to use a Drone to produce Potatoes and Farmer 2 uses a Drone to produce Potatoes, Farmer 1 will receive 50 and Farmer 2 will receive 50.
If Farmer 1 chooses to use a Drone to produce Tomatoes and Farmer 2 uses a Precision Tractor to produce Tomatoes, Farmer 1 will receive 30 and Farmer 2 will receive 25.

If Farmer 1 chooses to use a Drone to produce Potatoes and Farmer 2 uses a Precision Tractor to produce Tomatoes, Farmer 1 will receive 55 and Farmer 2 will receive 75.
Introductory Game Cont.

- If Farmer 1 chooses to use a Drone to produce Tomatoes and Farmer 2 uses a Precision Tractor to produce Potatoes, Farmer 1 will receive 75 and Farmer 2 will receive 55.

- If Farmer 1 chooses to use a Drone to produce Potatoes and Farmer 2 uses a Precision Tractor to produce Potatoes, Farmer 1 will receive 25 and Farmer 2 will receive 15.
Introductory Game Cont.

- If Farmer 1 chooses to use a Precision Tractor to produce Tomatoes and Farmer 2 uses a Drone to produce Tomatoes, Farmer 1 will receive 30 and Farmer 2 will receive 30.

- If Farmer 1 chooses to use a Precision Tractor to produce Potatoes and Farmer 2 uses a Drone to produce Tomatoes, Farmer 1 will receive 120 and Farmer 2 will receive 10.
Introductory Game Cont.

- If Farmer 1 chooses to use a Precision Tractor to produce Tomatoes and Farmer 2 uses a Drone to produce Potatoes, Farmer 1 will receive 20 and Farmer 2 will receive 110.

- If Farmer 1 chooses to use a Precision Tractor to produce Potatoes and Farmer 2 uses a Drone to produce Potatoes, Farmer 1 will receive 5 and Farmer 2 will receive 5.
Introductory Game Cont.

- If Farmer 1 chooses to use a Precision Tractor to produce Tomatoes and Farmer 2 uses a Precision Tractor to produce Tomatoes, Farmer 1 will receive 28 and Farmer 2 will receive 28.

- If Farmer 1 chooses to use a Precision Tractor to produce Potatoes and Farmer 2 uses a Precision Tractor to produce Tomatoes, Farmer 1 will receive 26 and Farmer 2 will receive 36.
If Farmer 1 chooses to use a Precision Tractor to produce Tomatoes and Farmer 2 uses a Precision Tractor to produce Potatoes, Farmer 1 will receive 36 and Farmer 2 will receive 26.

If Farmer 1 chooses to use a Precision Tractor to produce Potatoes and Farmer 2 uses a Precision Tractor to produce Potatoes, Farmer 1 will receive 48 and Farmer 2 will receive 48.
Introductory Game Cont.

- Come together and show each other your decision
- On a final piece of paper, each player should write down his or her name, which farmer they were, their decisions, and their final payoff
- Hand all pieces of paper in
Discussion

- How did your go about playing the game?
- What decision was made by each group of players?
- Was it a good choice for farmer 1 to reveal his choice?
Combined Sequential and Simultaneous Move Games

- Many games that you play have both a sequential and a simultaneous component to them.
- These games are typically more complex to analyze.
- To analyze these types of games, you need to combine the tools you used for solving sequential moves and simultaneous moves games.
Combined Sequential and Simultaneous Move Games Cont.

- You can have simultaneous move games that happen in sequential order
- You can have a simultaneous move game that will have implications on playing out different sequential moves
- Typically in these games you will have subgames that you will need to solve to get a full solution
Combined Sequential and Simultaneous Move Games Cont.

- In multi-level games, simultaneous games and sequential move games can be mixed and matched in any form.
- A subgame could provide a single payoff or multiple payoffs.
- Let’s examine our initial introductory game.
- Other examples of mixing and matching will be played in class.
Changing Simultaneous-Move Games into Sequential-Move Games

- There are times when you may be able to change a simultaneous-move game into a sequential-move game
- This can lead to four possibilities: a first-mover advantage, a second-mover advantage, both players ending better off, the same outcome
Example of First-Mover Advantage

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Advertise</th>
<th>Don't Advertise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertise</td>
<td>20, 20</td>
<td>10, 40</td>
</tr>
<tr>
<td>Don't Advertise</td>
<td>40, 10</td>
<td>5, 5</td>
</tr>
</tbody>
</table>
Example of Second-Mover Advantage

<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strawberries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lettuce</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strawberries</td>
<td>40, 45</td>
<td>95, 35</td>
</tr>
<tr>
<td>Lettuce</td>
<td>90, 25</td>
<td>70, 65</td>
</tr>
</tbody>
</table>
Example of Both Players Doing Better

<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strawberries</td>
<td>500, 350</td>
<td>200, 725</td>
</tr>
<tr>
<td>Lettuce</td>
<td>175, 475</td>
<td>125, 600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strawberries</th>
<th>Lettuce</th>
</tr>
</thead>
<tbody>
<tr>
<td>200, 725</td>
<td>125, 600</td>
</tr>
<tr>
<td>500, 350</td>
<td>175, 475</td>
</tr>
</tbody>
</table>
### Example of Neither Do Better

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Meat Sale</strong></td>
<td>Vegetable Sale</td>
</tr>
<tr>
<td></td>
<td>400 , 325</td>
</tr>
<tr>
<td><strong>Snack Sale</strong></td>
<td>300 , 550</td>
</tr>
</tbody>
</table>
Changing Sequential to Simultaneous

- Players can lose both first-mover and second-mover advantages when changed to simultaneous
- Both players could end up worse off by going to simultaneous-move games
Representing Simultaneous Move Games in Tree Form

- While we used the game matrix to represent simultaneous move games, it is possible to represent these games with a tree diagram.
- To do this, we have to use the idea of an information set.
- When the player does not know which node he/she is at, then we can draw a circle around the nodes or draw a line attaching them representing that the nodes are in the same information set.
Representing Simultaneous Move Games in Tree Form Cont.

- A strategy can now be defined as a complete plan of action which specifies a move that each player makes at a given information set which may contain multiple nodes.
- An information set can also be used to handle uncertainty.
- We have perfect information when we can associate each information set with a single node.
Example of Simultaneous Move Game Represented by a Tree

Player 1

Meat Sale

Snack Sale

Player 2

Vegetable Sale

Alcohol Sale

Vegetable Sale

Alcohol Sale

400, 325

625, 250

300, 550

500, 475
Showing Sequential Move Games as a Game Matrix

- It is possible to show a sequential-move game in a game matrix.
- When analyzing these games, you may find more Nash equilibrium than you have rollback equilibrium.
- Some of the Nash equilibrium can come from off-equilibrium paths of the tree.
- Credibility comes into play when threats are made to go off the equilibrium path.
Taking Sequential Move Game into a Game Matrix

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>VS,VS</th>
<th>AS,AS</th>
<th>VS,AS</th>
<th>AS,VS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat Sale</td>
<td></td>
<td>400, 325</td>
<td>625, 250</td>
<td>300, 550</td>
<td>500, 475</td>
</tr>
<tr>
<td>Vegetable</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td>500, 475</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vegetable</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alcohol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sale</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
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Final Discussion, Questions, and Thoughts