Simultaneous Move Games: Continuous Strategies, Discussion, and Evidence

Games Of Strategy
Chapter 5
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Terms to Know

- Best-response curve
- Best-response rule
- Continuous strategy
- Never a best response
- Quantal response equilibrium
- Rationalizability
- Rationalizable
- Refinement
Introductory Game

- The professor will have you count off.
- All individuals that have an even number will be on one side of the room and be known as the x players, while the odds will be on the other side of the room and known as the y players.
- Suppose that each of you would like to maximize your profits where $\pi = P*Q - cQ$ where P represents your price, Q represents the quantity demanded of your product, and c represents the unit cost of Q.
Introductory Game Cont.

- The quantity demanded for product x can be defined as $Q_x = 100 - 7P_x + P_y$ where $P_x$ is the price player x chooses and $P_y$ is the price player y chooses.

- The quantity demanded for product y can be defined as $Q_y = 100 - 7P_y + P_x$ where $P_y$ is the price player y chooses and $P_x$ is the price player x chooses.

- The cost of production for each player is 8.
Introductory Game Cont.

- Each side is allowed to work as a group
- On a piece of paper write the names of your teammates and the price your team will submit
Games Discussion

- How did your team figure out its best response to the other team?
Continuous Strategy Games

- There are games of strategy where the strategies that can be chosen are not necessarily discrete actions.
- This can make it difficult and challenging to try to represent the game in a matrix form.
- Even if the decisions are discrete in nature, e.g., price competition, there may be too many to deal with in a matrix.
- Hence we need to figure out a way to analyze games when the decisions are continuous or numerous.
Price Competition Example 1

- Suppose we have two players x and y whose each have a goal of maximizing profit.
- The quantity demanded for product x can be defined as $Q_x = 44 - 4P_x + 2P_y$ where $P_x$ is the price player x chooses and $P_y$ is the price player y chooses.
- The quantity demanded for product y can be defined as $Q_y = 104 - 4P_y + 2P_x$ where $P_y$ is the price player y chooses and $P_x$ is the price player x chooses.
Price Competition Example 1 Cont.

- Assume that the cost for each individual is $1 for each output Q produced
- Step 1: Set-up and solve each player's profit maximization problem
- Step 2: Find each player's best response function
- Step 3: Solve simultaneously the two players' best response function to find each's equilibrium
Price Competition Example 1 Cont.

- Step 4: Calculate the resulting output and profit at the equilibrium solution
- Solution will be done in class
Visual Representation of the Solution for Example 1
Collusion

- An agreement between rivals to work together
- Collusion can occur when there are a small number of players in a game
- When there are a small number of sellers, then it is said that there is an oligopoly; two sellers is known as a duopoly
Collusion Cont.

- When there are a small number of buyers, then it is said that there is an oligopoly.
- A cartel is an organization that tries to collude to raise prices.
What If The Two Competitors Could Collude in Example 1

- Suppose that player x and y decided that they were going to charge the same price through collusion.
- This would imply that $Q_x = 44 - 4P + 2P$ and $Q_y = 104 - 4P + 2P$ each with a cost of 1.
- What would be the optimal price?
Suppose we have two players $x$ and $y$ whose each have a goal of maximizing profit.

The quantity demanded for product $x$ can be defined as $Q_x = 42 - 6P_x + 6P_y$ where $P_x$ is the price player $x$ chooses and $P_y$ is the price player $y$ chooses.

The quantity demanded for product $y$ can be defined as $Q_y = 42 - 3P_y + 6P_x$ where $P_y$ is the price player $y$ chooses and $P_x$ is the price player $x$ chooses.
Price Competition Example 2 Cont.

- Assume that the cost for player x is $3 and the cost for player y is $6 for each output Q produced
- Solution done in class
- Draw the problem graphically
Cournot Quantity Competition

- Suppose there were two companies that competed based on quantity.
- Assume the demand curve for the product can be represented as \( P = 100 - 2Q \) where \( Q = q_x + q_y \), where \( q_x \) represents the amount that \( x \) would produce and \( q_y \) represents the amount \( y \) would produce.
- Suppose each face a cost of $10 for each unit each produces.
Cournot Quantity Competition

- What is each competitor’s profit function?
- What is each player’s best response function in terms of quantity?
- What is the equilibrium quantity that would be chosen by each competitor?
- What is the profit of each?
- What would be the profit if they could collude?
Bertrand Price Competition

- Assume the same demand function and cost from the Cournot game from before
- Instead of the two players competing on quantity, assume that they compete on price
- Further assume that if they each have the same price, then they will split the market evenly
- Otherwise, the competitor with the lower price gets all the consumers
- What is the equilibrium to this game?
Criticisms of Nash

- It does not take into account risk very well
- There can be multiple Nash equilibria
- The Nash equilibrium requires rationality

How did the textbook authors counter these claims?
Never a Best Response and Rationalizable

- This property is stronger in terms of eliminating a strategy than dominance of a strategy.
- You may be able to eliminate a strategy even though it is not dominated by using the argumentation of never a best response.
- Strategies are said to be rationalizable if they survive the property of never a best response.
Never a Best Response and Rationalizable Cont.

- If a game has a Nash, then it is rationalizable.
- Even if there are no Nash, there can be rationalizable outcomes.
- Rationalizability can take us to a Nash Equilibrium.
Example of Never a Best Response

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<th>P1S1</th>
<th>P1S2</th>
<th>P1S3</th>
<th>P1S4</th>
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Final Discussion, Questions, and Thoughts