Simultaneous Move Games: Discrete Strategies

Games Of Strategy
Chapter 4
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Terms to Know

- Assurance game
- Battle of the sexes
- Belief
- Best response
- Best-response analysis
- Chicken
- Convergence of expectations
- Coordination game
- Dominance solvable
- Dominant strategy
- Dominated strategy
- Focal point
Terms to Know

- Game matrix
- Iterated elimination of dominated strategies
- Mixed strategy
- Nash equilibrium
- Normal form
- Payoff table
- Prisoners’ dilemma
- Pure coordination game
- Pure strategy
- Strategic form
- Successive elimination of dominated strategies
Introductory Game

- Pair up with another individual
- Player 1 will be defined as the person whose last name is first in alphabetical order from A to Z; the other individual is player 2
- Each pair need to go to opposite sides of the room and not communicate with anyone
- Player 1 has three strategies—Up, Same, Down
- Player 2 has three strategies—Right, Center, Left
Introductory Game Cont.

- If Player 1 plays Up and Player 2 plays Right, then player 1 gets 55 and player 2 gets 40
- If Player 1 plays Up and Player 2 plays Center, then player 1 gets 45 and player 2 gets 75
- If Player 1 plays Up and Player 2 plays Left, then player 1 gets 35 and player 2 gets 50
Introductory Game Cont.

- If Player 1 plays Same and Player 2 plays Right, then player 1 gets 75 and player 2 gets 45
- If Player 1 plays Same and Player 2 plays Center, then player 1 gets 50 and player 2 gets 50
- If Player 1 plays Same and Player 2 plays Left, then player 1 gets 55 and player 2 gets 40
Introductory Game Cont.

- If Player 1 plays Down and Player 2 plays Right, then player 1 gets 45 and player 2 gets 10
- If Player 1 plays Down and Player 2 plays Center, then player 1 gets 40 and player 2 gets 60
- If Player 1 plays Down and Player 2 plays Left, then player 1 gets 25 and player 2 gets 15
Introductory Game Cont.

- Each player needs to choose a strategy from his/her strategy set and write it down on a piece of paper with the person’s name underlined and his/her opponent’s name.
- Each group of opponents needs to come together and figure out each of their payoffs based on the strategy chosen and put it down on his/her piece of paper.
- Hand it in.
Discussion

- What process did you use to choose your strategy?
- Was one strategy better than another?
- What kind of agribusiness example can you think of that might fit this game?
Strategies in Simultaneous Move Games

- Players have an opportunity to act only once, but the actions can be made up of multiple components.
- We can categorize how we think of strategies as either pure or mixed.
  - Mixed strategies are made up of a probabilistic combination of pure strategies.
- Strategies do not have to be discrete decisions.
Representation of Discrete Simultaneous Move Games

- These games can be represented in a game table/game matrix/payoff table
  - These are known as the normal form/strategic form of the game
- Depending on the number of players will dictate the dimensions of the table
Example of a Game Matrix

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy 1</strong></td>
<td><strong>Strategy A</strong></td>
</tr>
<tr>
<td>(Player 1 payoff, Player 2 payoff)</td>
<td>(Player 1 payoff, Player 2 payoff)</td>
</tr>
<tr>
<td><strong>Strategy 2</strong></td>
<td>(Player 1 payoff, Player 2 payoff)</td>
</tr>
</tbody>
</table>
Example 1

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Sell 1 Product</th>
<th>Sell 2 Products</th>
<th>Sell 3 Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raise Price</td>
<td>45 , 30</td>
<td>35 , 65</td>
<td>25 , 40</td>
</tr>
<tr>
<td>Same Price</td>
<td>65 , 35</td>
<td>40 , 40</td>
<td>45 , 30</td>
</tr>
<tr>
<td>Lower Price</td>
<td>35 , 0</td>
<td>30 , 50</td>
<td>15 , 5</td>
</tr>
</tbody>
</table>
Best Response

- In games, you want to look for your best response to other players’ strategies.
- Given a set of strategies chosen by the other players, a best response strategy is a strategy that you have that provides you with the best payoff with regards to the other strategies played.
- In the previous game, player 1’s best strategy to player 2 playing Sell 2 Products is Same Price.
Best Response Cont.

- We might write this as \( BR_1(\text{Sell 2 Products}) = \text{Same Price} \)
- In the previous example, what are the best response strategies for each player?
Nash Equilibrium

- A Nash equilibrium occurs when no player has an incentive to change his/her decision given that you hold all the other players’ chosen strategies constant.

- In Example 1, the Nash Equilibrium is Same Price, Sell 2 Products, i.e., \( BR_1(\text{Sell 2 Products}) = \text{Same Price} \) and \( BR_2(\text{Same Price}) = \text{Sell 2 Products} \).

- Nash Equilibrium does not imply that the payoff or your equilibrium choice is strictly greater than your other choices.
Nash Equilibrium

- There can be zero Nash equilibrium, one Nash equilibrium, or multiple Nash equilibrium.
- As long as payoffs are strictly greater in the equilibrium, the maximum amount of Nash equilibriums can be no larger than the number of strategies available to the player who has the least number or strategies.
- With simultaneous move games, you do not get to see the strategies that is played by your opponents, so you have to potentially rely on your beliefs of what they would do.
Example 2: Prisoners Dilemma

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Confess</th>
<th>Don't Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>-15 , -15</td>
<td>-2 , -30</td>
</tr>
<tr>
<td>Don't Confess</td>
<td>-30 , -2</td>
<td>-5 , -5</td>
</tr>
</tbody>
</table>

Player 2

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Confess</th>
<th>Don't Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>-15 , -15</td>
<td>-2 , -30</td>
</tr>
<tr>
<td>Don't Confess</td>
<td>-30 , -2</td>
<td>-5 , -5</td>
</tr>
</tbody>
</table>
Prisoners’ Dilemma

- In some games, the Nash equilibrium is suboptimal for all players in comparison to some other set of strategies.
- The Prisoners’ Dilemma arises because of dominant and dominated strategies.
Dominant versus Dominated Strategy

- A strategy $s_1$ is said to “dominate” a strategy $s_2$, when all the payoffs from playing strategy $s_1$ are preferred to their corresponding payoffs in strategy $s_2$.
- A strategy is a dominated strategy if there is one strategy in your set of strategies that dominates it.
- A strategy is a dominant strategy if it dominates all other strategies you have available to you.
Dominant versus Dominated Strategy Cont.

- A dominant strategy will always be played, whereas a dominated strategy will never be played.
- If a dominant strategy exists, then a Nash equilibrium will be in it.
- If a strategy is dominated, then it cannot have a Nash equilibrium.
Successive/Iterated Elimination of Dominated Strategies

- When a strategy is dominated, we know that the strategy will not be played.
- Based on rationality, your opponent knows you will not play your dominated strategy.
- Since dominated strategies do not get played, you can remove them from your set of strategies; hence, creating a “new” smaller game.
Successive/Iterated Elimination of Dominated Strategies Cont.

- By eliminating dominated strategies and creating smaller games, you can potentially find Nash equilibrium.
- If you get a unique solution from this method, the game is known as dominance solvable.
- If certain strategies are “weakly” dominated, this method can eliminate Nash equilibrium.
Best Response Analysis

- This type of analysis identify which of your strategies are best to choose in comparison to each strategy the other players could possibly play.

- Considering a two person game, a Nash equilibrium occurs when \( \text{BR}_1(s_2) = s_1 \) and simultaneously \( \text{BR}_2(s_1) = s_2 \), where \( \text{BR}_i \) represents the best response function for player \( i \) and \( s_i \) represents player \( i \)'s strategy played.
Three Player Simultaneous-Move Games

- When there are three players in a game, you can represent the third player by adding another set of matrices that represent each of the strategies of the third player.
- You will need to add a third payoff to the set of matrices.
- Payoffs for the third player will have to be compared across matrices.
Example 3: Three Player Game (Player i’s payoff is in the i-th position)

Player 3: Forward

<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Up</td>
<td>Left</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7,7,7</td>
</tr>
<tr>
<td>Down</td>
<td></td>
<td>8,5,5</td>
</tr>
</tbody>
</table>

Player 3: Back

<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Up</td>
<td>Left</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5,5,8</td>
</tr>
<tr>
<td>Down</td>
<td></td>
<td>6,3,6</td>
</tr>
</tbody>
</table>
Multiple Equilibria Games

- Many games have more than one equilibrium
- Some of the games are called coordination games
- In coordination games, each player wants to make sure that they choose the same strategy, e.g., go to the same movie, sign-up for the same class, etc.
- Coordination games are helped if there is a focal point
Multiple Equilibria Games Cont.

- Assurance games have a focal point strategy.
- Battle of the sexes games have multiple equilibrium where one player prefers one of the equilibrium and the other player prefers a different equilibrium, but each would prefer to choose the same strategy over differing strategies.
- Chicken games are where there are multiple equilibrium and each player does not want to choose the same strategy as the other.
Final Discussion, Questions, and Thoughts