Games with Sequential Moves
Games Of Strategy
Chapter 3
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Terms to Know

- Action node
- Backward induction
- Branch
- Decision node
- Decision tree
- Equilibrium path of play
- Extensive form
- First-mover advantage
- Game tree
- Initial node
- Intermediate valuation function
- Move
Terms to Know

- Node
- Path of play
- Prune
- Rollback
- Rollback equilibrium
- Root
- Second-mover advantage
- Terminal node
Introductory Game

- Split up into groups of two
- Play a game of rock, paper, scissors until one of you wins
- The person who wins gets to choose whether he/she moves first or second
- Please write down on a piece of paper who is moving first and who is moving second
Introductory Game Cont.

- The first mover gets to choose between Up and Down which should be written down on a piece of paper.
- The second mover must choose between Right and Left if the first mover chooses Up.
- The second mover must choose between Enter and Exit if the first mover chooses Down.
Introductory Game Cont.

- If the first mover chooses Up and the second mover chooses Right, then the first mover receives 45 and the second mover receives 55
- If the first mover chooses Up and the second mover chooses Left, then the first mover receives 55 and the second mover receives 45
- If the first mover chooses Down and the second mover chooses Enter, then the first mover receives 40 and the second mover receives 60
- If the first mover chooses Down and the second mover chooses Exit, then the first mover receives 60 and the second mover receives 40
Introductory Game Cont.

- Please submit the piece of paper with your names, the decision each of you made, and the score for each
Discussion

- What was important to you?
- How did you come to your decision?
- Did anyone have an advantage?
- How could this be related to agribusiness?
Sequential Move Games

- They have a strict sequence of play
- A player makes moves with the knowledge of what previous players have done
- Players need to take into account how their decisions will influence others’ decisions
- Need to calculate future consequences
Game Tree

- Also known as an extensive form of the game
- It is made up of players, actions, and payoffs
- One component of a game tree is a node (initial node, action node/decision node)
- Nodes allow for keeping track of who/what is making a decision
- Game trees also have branches which connect nodes and represent actions/decisions made
Game Tree Cont.

- There are special nodes that exist to represent when random/probabilistic acts occur
  - These nodes are said to be “Nature” as the player
- An ending node, known as a terminal node represents the final action or decision made and is associated with the payoff for each player
- A path of play represents the actions made by all players from an initial node to a terminal node connected by a set of branches action nodes
Game Tree Cont.

- Payoffs at the terminal node are typically listed by the order of which the players made their first move.
  - You should make sure to explain the order of payoff rather than assuming someone knows.
- A strategy in a game tree represent all the moves a person could potentially make in the whole game.
  - It must account for all eventualities.
  - It is made up of all decision nodes a player could conceivably reach.
Simple Example of a Game Tree

Player 1

- Action 1
  - Action 3
    - Player 2
      - Action 4
        - Player 1 Payoff, Player 2 Payoff
      - Action 5
        - Player 1 Payoff, Player 2 Payoff
    - Action 6
      - Player 1 Payoff, Player 2 Payoff
Simple Example of a Game Tree with Nature

Player 1
- Action 1
- Action 2

Player 2
- Action 3
- Action 4
- Action 5
- Action 6

Nature
- Event 1, Prob = A
- Event 2, Prob = B

Player 1 Payoff, Player 2 Payoff
Player 1 Payoff, Player 2 Payoff
Player 1 Payoff, Player 2 Payoff
Player 1 Payoff, Player 2 Payoff
Player 1 Payoff, Player 2 Payoff
Player 1 Payoff, Player 2 Payoff
Solving Sequential Games

- To solve these games, you need to use a method known as the rollback method.
  - This method has you start examining the optimal decisions at the terminal nodes first and working your way to the initial node.
  - This is also called backward induction or the fold back method.
- When all players use the rollback method to find their optimal strategies, then you would get a rollback equilibrium or subgame perfect equilibrium.
Example 1

Player 1

Harvest Monday

Harvest Tuesday

Player 2

Harvest Wednesday

Harvest Thursday

Harvest Thursday 2

Harvest Friday

60, 30

45, 45

70, 20

40, 50
Questions for Example 1

- What are the strategies for each player?
- What does each player do?
- Represent the players payoffs for each strategy.
Example 2: Change Payoffs to Rankings

Player 1
- Harvest Monday
- Harvest Tuesday

Player 2
- Harvest Wednesday (2, 3)
- Harvest Thursday (3, 2)
- Harvest Thursday2 (1, 4)
- Harvest Friday (4, 1)
Example 3: Adding Another Move

Player 1
- Lower Price
  - Player 2
    - Keep Prices Constant
      - Not Advertise
        - Advertise
          - Player 1
            - 120, 30
            - 100, 50
    - Raise Price2
      - 140, 10
      - 60, 90
    - Keep Prices Constant2
      - Raise Price3
        - 75, 75
Order Advantage: First Mover versus Second Mover

- Does order of movers matter?
- Is it always best to be the first mover?
- Is it always best to be the second mover?
Example 4: First-Mover Advantage

Player 1
- Adopt Drones
  - Player 2
    - Adopt Drones2
      - 500K, -100K
    - Do Not Adopt Drones2
      - 750K, -10K
      - -10k, 250K
  - Do Not Adopt Drones
    - Adopt Drones3
      - 0, 0
    - Do Not Adopt Drones3
Example 4: Second-Mover Advantage

Player 1
- Adopt Drones
- Do Not Adopt Drones

Player 2
- Adopt Drones2
  - -50K, 100K
- Do Not Adopt Drones2
  - -90K, -10K
- Adopt Drones3
  - -80, 50K
- Do Not Adopt Drones3
  - -120K, 20K
Second Game

- Split up into groups of two
- Play a game of rock, paper, scissors until one of you wins
- The person who wins gets to choose whether he/she moves first or second
- Please write down on a piece of paper who is moving first and who is moving second
The first mover is allocated 100 points that he/she must split between the two players. The second-mover gets to decide whether to accept the offer or decline the offer. If the offer is accepted then each player gets the proposed split. If the offer is rejected, then both players get nothing. On a piece of paper, write down the results and hand it in.
Third Game

- You will be allocated a number by the professor from 1 to the number of individuals in the class.
- If you are an even number put a number from 0 to 100 on a piece of paper along with your number.
- The professor will collect these pieces of paper and randomly hand them out to individuals who have odd numbers.
Third Game Cont.

- The odd numbered individuals will write their number on the paper along with accept or decline where accept means that you agree with the split given to you.
- These individuals will hand the paper back to the professor.
Fourth Game

- Pick a partner
- The taller person will be denoted odd and the shorter individual will be denoted even, ties in height will be decided by rock, paper, scissors with the winning being denoted as odd
- Write down on a single piece of paper whether you are odd or even
- Next, behind your back select either one finger or two on your right hand while not showing your partner
Fourth Game Cont.

- On the count of three from the professor, show your partner the number of fingers you chose on your right hand.
- If the number is odd, then the odd person will become the first mover while if the number is even then the even person becomes the first mover.
- Write down who is the first mover and second mover.
- The first-mover is allocated 10 points.
Fourth Game Cont.

- If the first-mover (player 1) says stop he/she gets 10 points and the second-mover (player 2) gets nothing and the game ends.
- If the first-mover (player 1) says continue, then 10 points gets added to the original 10 points and now player two gets the 20 points and player 1 gets nothing.
- Player 2 now has the option to say stop and keep the 20 points, or say continue and pass 30 points to player 1.
- If no one says stop, then this continues until 50 is reached.
- Once 50 is reached, the game ends.
Fifth Game

- Swap who is first mover
- Game four will be played again with one change
- The end of the game will occur based on a random draw after the third iteration, i.e., if the random number is odd, then the game stops
Final Discussion, Questions, and Thoughts