Linear Programming: Basic Concepts

Chapter 2: Hillier and Hillier
Agenda

- Define Linear Programming
- The Case of the Wyndor Glass Co.—A Maximization Problem
- Developing a Mathematical Representation of Wyndor’s Problem
- A Spreadsheet Model of Wyndor’s Problem
- A Graphical Solution to Wyndor’s Problem
A Spreadsheet Solution to Wyndor’s Problem

The Case of Profit & Gambit—A Minimization Problem
Linear Programming (LP)

- Linear programming can be defined as the planning of activities represented by a linear mathematical model.
  - Hence, in a linear programming problem, the objective and the constraints are all linear.
  - Given this assumption of linearity, LP is used to find the best mix of activities.
A Summary of the Case of Wyndor Glass

- Two new products have been developed:
  - An 8-foot glass door
  - A 4x6 foot glass window
- Wyndor has three production plants
  - Production of the door utilizes Plants 1 and 3
  - Production of the window utilizes Plants 2 and 3
- Objective is to find the optimal mix of these two new products.
### A Summary of the Case of Wyndor Glass Cont.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Doors</th>
<th>Windows</th>
<th>Available Per Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 hour</td>
<td>0 hour</td>
<td>4 hour</td>
</tr>
<tr>
<td>2</td>
<td>0 hour</td>
<td>2 hour</td>
<td>12 hour</td>
</tr>
<tr>
<td>3</td>
<td>3 hour</td>
<td>2 hour</td>
<td>18 hour</td>
</tr>
<tr>
<td>Unit Profit</td>
<td>$300</td>
<td>$500</td>
<td></td>
</tr>
</tbody>
</table>
A Mathematical Representation of Wyndor’s Problem

- In general terms, we need to do five steps when developing the mathematical model:
  - Gather the relevant data
    - This is from the table on the previous slide
  - Identify the decisions to be made, i.e., decision variables
    - The amount of doors (D) and windows (W) to produce
A Mathematical Representation of Wyndor’s Problem Cont.

– Identify the constraints of the decisions

• There are five constraints:
  – How much of Plant 1 the doors and windows use
  – How much of Plant 2 the doors and windows use
  – How much of Plant 3 the doors and windows use
  – The two non-negativity constraints on the quantity produced of doors and windows
A Mathematical Representation of Wyndor’s Problem Cont.

– Identify the overall measure of performance, i.e., the objective function
  • Profit = $300*(#Doors) + $500*(# Windows)

– Convert the verbal description of the constraints and the measure of performance into quantitative expressions in terms of the data and the decisions
  • Given on next slide
A Mathematical Representation of Wyndor’s Problem Cont.

Max $300D + 500W$

w.r.t. D, W

Subject to:

D $\leq$ 4 (Constraint 1)

2W $\leq$ 12 (Constraint 2)

3D + 2W $\leq$ 18 (Constraint 3)

D $\geq$ 0, W $\geq$ 0 (Constraints 4 and 5)
A Spreadsheet Model of Wyndor’s Problem

- Formulation Procedure
  - Gather data for the problem
  - Enter the data into data cells on the spreadsheet
  - Identify the decisions to be made on the level of the activities and designate them as the changing cells for displaying these decisions
  - Identify the constraints on the decisions and introduce output cells as needed to specify the constraints
A Spreadsheet Model of Wyndor’s Problem Cont.

– Choose the overall measure of performance to be entered into the target cell.

– Use an appropriate Excel function, e.g., the SUMPRODUCT function, to enter the appropriate value into each output cell (including the target cell).
A Spreadsheet Model of Wyndor’s Problem Cont.

- See Hillier and Hillier CD spreadsheet for chapter 2.

- Two special features are in this spreadsheet model:
  - Naming of a range of cells
  - SUMPRODUCT Excel Command
Naming a Range of Cells

- To Name a range of cells, you must first select the cells you would like to name.
- Once these cells are selected, you want to choose insert from the main menu.
- Next choose Name and then Define.
- When the Dialog box comes up, type in the name you would like to call the range of cells.
- Finally, click close.
SUMPRODUCT Function in Excel

- The SUMPRODUCT function multiplies corresponding numeric components in given ranges or arrays and returns the sum of those products.
- An array in this case is a list of numbers where the ordering of the list and the dimensionality (number of elements in the list) of the list matters.
  - Example: 1, 2, 3, 4, 5 is an array
  - Example: 5, 4, 3, 2, 1 is a completely different array
- Arrays can be known as column or row vectors.
SUMPRODUCT Function in Excel

- Suppose you have two arrays of numbers.
  - Array1 is defined as a1, a2, a3
  - Array2 is defined as b1, b2, b3
- \( \text{SUMPRODUCT}(\text{Array1}, \text{Array2}) = a1b1 + a2b2 + a3b3 \)
- Suppose
  - Array1 is defined as 1, 2, 3
  - Array2 is defined as 3, 2, 1
- \( \text{SUMPRODUCT}(\text{Array1}, \text{Array2}) = 1 \times 3 + 2 \times 2 + 3 \times 1 = 10 \)
Needed Terminology for Graphical Solution

- **Constraint Boundary Equation**
  - This equation dictates the boundary of the feasible and infeasible solutions given a particular constraint.

- **Constraint Boundary Line**
  - This line is dictated by the constraint boundary equation when you have only two decision variables.
  - It is the edge of the constraint boundary equation when the constraint is met with equality.
Needed Terminology for Graphical Solution Cont.

- **Objective Function Line**
  - This line is a line determined by setting the objective function to a particular value.
  - Each point on this line gives the same value for the objective function.

- **Feasible Solution**
  - A point within the region that consist of all the points that can be considered as a solution.
  - It is a point that meets all the boundary constraints simultaneously.
Needed Terminology for Graphical Solution Cont.

- **Infeasible Solution**
  - A point that does not meet all of the constraints.

- **Optimal Solution**
  - A point or points that optimize the objective function.

- **Feasible Region**
  - This is the region that holds all the feasible solutions.
Graphical Method of Solving Wyndor’s Problem

- This will be examined in class using the LP Interactive Management Module located at http://highered.mcgraw-hill.com/sites/0073129038/student_view0/management_science_modules.html, as well as solving this by hand.
  - What happens when you change 300 to 3 and 500 to 5?
  - What happens when you change D from being less than 4 to now being less than 5?
Needed Terminology for Solver Solution

- **Changing Cells**
  - These are the cells on the spreadsheet that are representative of the decision variables.

- **Data Cells**
  - These are the cells on the spreadsheet that show all the parameters/data of the problem.
Needed Terminology for Solver
Solution Cont.

- **Output Cells**
  - These are cells that are calculated using the changing cells/decision variables.

- **Target Cell**
  - This cell represents the overall measure of performance, i.e., the objective function.
Major Components of Solver

- Set Target Cell
- Equal to:
  - Max
  - Min
  - Value of
- By Changing Cells
Major Components of Solver

- Subject to the Constraints
  - Add
  - Change
  - Delete
- Options
  - Assume Linear Model
  - Assume Non-Negative
- Solve
Solver Method of Solving Wyndor’s Problem

- This will be examined in class using Solver.
A Summary of the Case of Profit & Gambit

- Trying to develop an advertising campaign that will affect their three products.
  - Stain Remover
  - Liquid Detergent
  - Powder Detergent

- They have a choice of:
  - TV
  - Print media
A Summary of the Case of Profit & Gambit Cont.

- Profit & Gambit has three restrictions on the minimum required increase on product sales.
- Objective is to find the optimal mix of these two advertising avenues that meet the constraints at the lowest cost.
### A Summary of the Case of Profit & Gambit Cont.

<table>
<thead>
<tr>
<th>Product</th>
<th>Increase in Sales Per Unit of Advertising</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TV</td>
</tr>
<tr>
<td>Stain Remover</td>
<td>0 %</td>
</tr>
<tr>
<td>Liquid Detergent</td>
<td>3%</td>
</tr>
<tr>
<td>Powder Detergent</td>
<td>-1 %</td>
</tr>
<tr>
<td>Unit Cost</td>
<td>$1 Mil.</td>
</tr>
</tbody>
</table>
A Mathematical Representation of Profit & Gambit Cont.

- In general terms, we need to do five steps when developing the mathematical model:
  - Gather the relevant data
    - This is from the table on the previous slide
  - Identify the decisions to be made, i.e., decision variables
    - The amount of television ads run (TV) and the amount of print ads run (PM)
A Mathematical Representation of Profit & Gambit Cont.

– Identify the constraints of the decisions

• There are five constraints:
  – A minimum increase in the amount of sales of the Stain Remover
  – A minimum increase in the amount of sales of the Liquid Detergent
  – A minimum increase in the amount of sales of the Powder Detergent
  – The two non-negativity constraints on the quantity of TV and Print Media ads
A Mathematical Representation of Profit & Gambit Cont.

– Identify the overall measure of performance, i.e., the objective function
  • Cost = $1 Mil.*(# TV) + $2 Mil* (# PM)

– Convert the verbal description of the constraints and the measure of performance into quantitative expressions in terms of the data and the decisions
  • Given on next slide
A Mathematical Representation of Profit & Gambit Cont.

Min \( TV + 2PM \)
\( \text{w.r.t. TV, PM} \)
Subject to :
\( PM \geq 3 \) (Constraint 1)
\( 3TV + 2PM \geq 18 \) (Constraint 2)
\( -1TV + 4PM \geq 4 \) (Constraint 3)
\( TV \geq 0, PM \geq 0 \) (Constraints 4 and 5)
Solution to Profit & Gambit

- The following solutions will be discussed in class:
  - Hand
  - Graphical
  - Solver